A Case Study of A Mathematics Teacher's Pedagogical Values: Use of A Methodological Framework of Interpretation and Reflection

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Abstract

The impact that mathematics teachers' pedagogical beliefs, whether consistent or not, have on their classroom instructional practices and student conceptions have been well documented. A probable source for the (in)consistencies of such beliefs is considered to be teachers' mathematical and pedagogical values as expressed in the deeper affective and evaluative qualities that underpin mathematics teachers' pedagogical preferences, judgments, and choices. Drawing on the social-psychological theories of Vygotsky, Lave, and Tajfel, values in this paper are conceived as a dual individual/social phenomenon, a personal/social identity concerning mathematics and pedagogy. Case study methods of investigation, involving classroom observation, questionnaire survey, and interview discussion, were used to explore one senior mathematics teacher's pedagogical values. This paper reports some interim results based on the first year of a three-year research project funded by the National Science Council of the ROC.

The teacher's pedagogical values were interpreted in terms of three phases, in which each phase consisted of the five components of social, educational, mathematical, mathematics educational, and pedagogical aspects. The Intention Phase describes what he said before instruction. The Implementation Phase reveals the teacher's mathematics and pedagogical values during instruction. The Self Phase then integrates the values attached to the previous two phases into a system which consists of a group of five pedagogical identities (core values). This "invisible inner value-laden pedagogical self" acts as the controller in the teacher's thinking and instruction.

Conceiving values from a sociological point of view, we suggest that it is necessary for researchers to examine teachers' pedagogical values by using "a methodological framework of investigation and interpretation". Three aspects of the framework are the investigative approaches used, the characteristics examined, and the subsequent stages considered in interpreting values. Two processes used to define valuing, "acting" and "choosing", were helpful in examining teachers' pedagogical values. Values were construed in this study as the principles or standards of teacher choices and judgements on the importance or worth of using certain pedagogical identities in classroom teaching of mathematics. A value system was conceived as "the internal-external dialectic of identification", or the process whereby all values are constituted.

Key Words: Value, pedagogical value, identity, pedagogical identity

I. Background and Problem

The format of classroom mathematics teaching and the directions of mathematics educational reform in Taiwan have been enormously influenced by the American ethos. For instance, the idea of constructivism and the four central ideas underpinning primary school mathematics curriculum--the conception of mathematics as problem solving, reasoning, communication, and mathematical connections--are the same as what the American curriculum stresses (Chou, 1994; Ning, 1993; NCTM, 1989, 1991). These ideas also play the crucial role in the implementation of the New Mathematics Curriculum for primary and secondary schools (ME, 2000, p.135). Should we also accept unquestioningly the educational values underpinning such an ethos? for school mathematics curriculum is conceived as a carrier of values (Bishop, 1988; Eson, 1964; Frondizi, 1970). Studies have shown many negative phenomena that occur in mathematics classrooms, for example, mathematics is nothing more than a set of formula and symbols which is not practical, and must be learned through memorization and drills. These negative value phenomena may be connected with social-educational needs and the nature of mathematical knowledge (Bishop, 1991; Swadener & Soedjadi, 1988). Therefore, we ought to think more carefully about questions of values concerning the mathematics and pedagogy classroom teaching conveys and the pedagogical values students have been taught through the teaching of mathematics, because these questions have been largely ignored by researchers in mathematics education (Bishop, 1991).

Values have been conceived as personal experiences, objects of thought, or psychological phenomena (Frondizi, 1970); individual feelings (Meinong, 1894) or objects to be desired (Scheler, 1954). In the educational context, we consider values as psychological phenomena reflecting personal experiences, relating to and situated in broader social-cultural contexts. "Values" refer to individual principles of selection and judgement (COBUILD, 1990; Samuel, 1937); or ideas or concepts concerning the worth of something (Swadener & Soedjadi, 1988). On the other hand, Tajfel's (1978, 1981) theory of social identities describes collective cognition, such as beliefs and values, in terms of personal characteristics and emotions attached to a certain social group. As social and individual phenomena, values are therefore conceived in this paper as a teacher's pedagogical identities concerning mathematics, teaching, learning, and the curriculum. They reveal the principles or standards of each teacher's choices and judgements concerning the importance or worth of using certain pedagogical identities in his or her classroom teaching of mathematics.

A domain of research relevant to values is teachers' beliefs. On one hand, mathematics teachers may hold various pedagogical beliefs, which may be different in form as mathematical or pedagogical, or in level as enacted or espoused; nevertheless, beliefs have to do with an individual's preference for certain identifications concerning mathematics and pedagogy. On the other hand, values are more about personal principles of choosing and judging across such identifications or preferences. Values are conceptualized as the deep affective qualities which teachers promote and foster through the subject of mathematics; their relationship with beliefs was summed up by Bishop and Clarkson (Bishop, 1999) in the phrase "values are beliefs in action". In other words, a teacher may hold various beliefs that become values when enacted in his or her classroom teaching.

In this paper, we identified and interpretd the pedagogical values of one mathematics teacher through a "Three Phases (Intention, Implementation, and Self) and Five Components (Social, Educational, Mathematical, Mathematics educational, and Pedagogical)" framework. The methodologies used in this study were re-considered and re-structured.

II. Research Methods

The case study method including questionnaire surveys, interviews, and classroom observations was used to explore one senior teacher's (T1) mathematics and pedagogical values. The teacher had a master's degree in mathematics and had taught mathematics in a public senior high school for 20 consecutive years. In general, the type of classroom teaching was "teacher stands in front of the class and talks to the students while all students work together". His teaching patterns revealed consistent salient features. "Dialogue interviews" including reflective and introspective discussion, and recursive probing procedures were developed and used in the interviews, in which the teacher played an active role in the conversation while the researcher acted as a listener and inquirer. A set of probes was also used in such dialogues: For instance, "What would you consider to be the most important things in your teaching? And why?", "What kinds of messages did you try to pass on to your students through mathematics teaching? And why?", and "For what reasons did you teach mathematics like this?"

A two-facet observation system focussing on features of the teaching activities, behaviors, and intentions of the teacher was developed to describe T1's teaching. The first facet included format of teaching such as metaphors or demonstrations; content of teaching such as stories or mathematical problems; and methods of teacher-student interaction such as teacher explanations or teacher-student dialogues. The second facet included patterns of teacher behavior, such as the description of phenomena or demonstration of an example, and the intention of the teacher such as investigation of phenomena or phenomenon-concept connection.

We then used certain critical events derived from T1's teaching as probes for further interviews. Using the processes of "Retrospection" and "Introspection", it was assumed that the teacher's identities of mathematics and pedagogy might become explicit, and hopefully point towards the pedagogical values expressed in his teaching. The teaching of three topics were videotaped, including "Mathematical induction," "Circle," and "Permutations"; these consisted of five, three, and five lessons respectively. Another senior secondary mathematics teacher acted as an independent checker to examine the reliability of the observational system using Cohen's Kappa (1960). As a result, the Kappa coefficients showed a very high consistency over the categories. We then used the system to analyze each observed lesson.

Videotapes of five experienced teachers, including a colleague (T2) of T1 and four teachers (A, B, C, D) who taught mathematics at other senior high schools, were used as the catalysts for uncovering and re-examining T1's values. T2 and two student teachers (ST1, ST2) attached to T1's school were invited to be independent checkers to re-examine our judgements. In the following section, we will report some major results from the first year of this three-year research project on the values of mathematics teachers.

III. Interpretation of Teachers' Pedagogical Values

Based on the analysis of T1's classroom teaching activity, assumptions concerning pedagogical identifications were uncovered so that they might point to the teacher's principles of choosing and judging with regard to those identifications. Using interview data after the lessons, we revised or confirmed the assumptions and identified the resulting central principles concerning the implementation phase of pedagogical values. Using those interview data before the lessons, we described the teacher's principles for designing the lessons attached to the intention phase of pedagogical values. We then identified several core pedagogical principles in the self phase of pedagogical values; these principles were consistent with the previous two phases. Finally, T1's pedagogical value system was proposed and discussed. The empirical data referred to here are mainly derived from the topic of mathematical induction.

1. Teaching Practice

The "Hanoi Tower" activity was re-framed by T1 as a basis for introducing the concept of mathematical induction. This initiating activity took about 15 minutes in the first lesson, during which time the teacher's demonstration and teacher-student dialogue were as shown in Table 1. The two critical questions used by the teacher to get the students to think were:

(1) Can you do it?

(2) Do you believe that if N=3 is possible then N=4 will also be possible?

A brief transcript of the last part of the teaching sequence listed in Table 1 is used to summarize T1's classroom teaching activities:

(T1, 971021, video transcript)

T1: S2 has done very well, however, S3 failed, let me show you the way I prefer. First of all, can you do it if the number is 3?

Si (whole class replies): Yes, of course we can.

T1: If the number is 4, could I pack it up as a unit and move the package from A to B?

Si: It is okay.

T1: Then, if I move the fourth one to C, is it okay?

Si: Yes.

T1: Then if I move this package again from B to C, can I do it?

Si: Oh! No, you are cheating.

S3: My goodness! What if I do it the way that you just showed us by packing up the case of 4 to solve the case of 5, and then solve the case of 6, and so on?

T1: Are you sure?

S3: Why'not?

T1: Excellent (he smiles expressively), are you convinced that I didn't cheat you?

Si: Yes! There should not be any problem.

T1: Why you are so sure about it?......

Si: Since the case of three is done.

T1: Right, how about 4?

Si: There shouldn't be any problem.

T1: Okay, how about 5?

Table 1. The Hanoi Tower Activity in the Mathematical Induction Topic

Sequence of Teaching	Duration	Type of Activity
T1: (Teacher's demonstration and students' response)	3'30"	Demonstration
Si: (Whole class discussion)	1'	Dialogue
S1: (Student manipulation, N=3, Success)	1'	Dialogue
T1: (Teacher explanation only)	1'30"	Demonstration
Si: (Whole class discussion, N=4)	2'	Dialogue
S2: (Student manipulation, N=4, Success)	2'	Dialogue
S3: (Student manipulation, N=4, Failure)	1'	Dialogue
T1&Si: (Teacher explanation and student response)	3'	Dialogue
Tota	15'	

Si: It can be done similarly if we solve it first by 4.
T1: Right, how about 10?
T1: How about 100? --- 1000? --- 10000? How can you do it? S4, could you say something about it?
S4: If 3 is ok, then 4 can be done, and 5 can also be done, and then for 6, ----, for 10, for 100, for 1000, and so on. Anyway, you just do it by the same method.
T1: How about 1000000?
Si: It can also be done by the same method. There will not be any problem.
T1: Therefore, we can do it all the way through in using the same method?
Si: Yes, why not.
T1: In this case, are you convinced that for any countable number we can always do it this way?
Si: Yes, we can do it by counting up.

The above example will serve as the background for discussion of the two selected implemented values in the following section.

The Hanoi Tower problem was used in this section as a teaching aid for developing the essential concept of mathematical induction. It was followed by the "coloring problem" activity concerning the number of sections made by bisecting a plane by n different lines. Here the problem was presented by asking students the question "Can you do it if the plane is bisected by adding one more line, and each pair of bisecting areas must have different colors (either black or white)?". At the end of the first lesson, the teacher then gave a familiar example and asked the students to solve it. The formal assumptions of mathematical induction were introduced in the second lesson after the example was discussed. An effective way of introducing mathematical induction seems to be to introduce the concept through an explanation by the teacher, and then perhaps use less than 5 minutes to focus on the two steps of the principle of induction.

However, why did T1 choose to introduce the concept in this way? And what were the principles by which he judged and chose to use such a teaching approach? In the following sections we describe the pedagogical values connected with the activity. Moreover, other pedagogical values related to the activity are uncovered, and the pedagogical value system of T1 are discussed.

2. Pedagogical Values

A set of 20 different but related pedagogical values that T1 had were grouped in terms of a "Three Phases-Five Components Framework", consisting of the self, intention, and implementation phases, and the

Table 2. A Three Phases and Five Components Analysis of T1's Pedagogical Values

Phase / Component	Self	Intention	Implementation
Social	● The evolution of culture and society is led by the elite.	● Teachers have to create opportunities for students in order to facilitate cultural development.	● Teachers should wait and identify the few elite students who can enable social and cultural development.
Educational	● Education seeks to empower one's abilities and qualities, and to improve the qualities of human life.	● Education seeks to empower one's abilities and qualities.	● Education seeks to empower one's abilities and qualities, and to make life happier. ● Education seeks to educate people's knowledge as a whole.
Mathematical	● Mathematics is a useful and interesting subject.	● Mathematics is an interesting and useful subject. ● Mathematics is a practical, abstract, and ideal subject.	● Mathematics is an interesting and useful subject that should be acquired with pleasure. ● Mathematics is a practical, abstract, and ideal subject.
Mathematics Educational	● Mathematics education seeks to develop students' knowledge, abilities, and intellect.	● Mathematics education seeks to improve students' knowledge and abilities.	● Mathematics education seeks to improve students' involvement and to like mathematical knowledge. ● Mathematics education seeks to develop students' knowledge, abilities, intellect, and personalities.
Pedagogical	● Mathematics teaching is an activity to initiate desire, expectations, and enjoyment of knowledge.	● Mathematics teaching is an activity to increase students' motivation and anticipation for learning.	● Mathematics teaching seeks to teach students the nature of mathematical knowledge rather than mathematical forms. ● Mathematics teaching seeks to motivate students' interest and willingness to learn.

social, educational, mathematical, mathematics educational, and pedagogical components. The four values attached to the pedagogical components, as shown in Table 2, are exemplified and discussed. The remaining values were discussed elsewhere (Chin & Lin, 1999). The designation of the data (teacher interviewed, date of the interview, before or after teaching session) refers to relevant information concerning the data. For example, (T1, 971027, after mathematical induction) stands for the interview conducted on 27th of October, 1997, following the lessons of mathematical induction.

There were two pedagogical values T1 attached to and implemented in the "Hanoi Tower" activity:

A. *Mathematics teaching seeks to teach students the nature of mathematical knowledge rather than mathematical forms:*

The use of the Hanoi Tower activity helps students learn the fundamental ideas underpinning the concepts of mathematical induction. Having collected the teacher's dialogue in the previous short transcript, we find that values of the processes of "conjecture", "induction", "re-confirmation", and "conviction" concerning the substance of the concept of mathematical induction are implicitly transferred through this teaching activity.

(T1: 971021, edited from the video transcript)

T1: S2 has done very well, however, S3 failed, let me show you the way that I prefer. First of all, *can you do it* if the number is 3?

T1: If the number is 4, could I pack it up as a unit and move the package from A to B?

T1: Then, I move the fourth one to C, is it okay?

T1: Then if I move this package again from B to C, *can I do it*?

T1: *Are you sure*?

T1: Excellent (he smiles expressively). Are you convinced that I didn't cheat you?

T1: Why you *are so sure about it*?---

T1: Right, how about 4?

T1: Okay, how about 5?

T1: Right, how about 10?

T1: How about 100? --- 1000? --- 10000? How can you do it? S4, could you say something about it?

T1: How about 1000000?

T1: Therefore *can we do it* all the way through in using the same method?

T1: In this case, are you convinced that for any countable number *we can always do* it this way?

In the interviews focusing on this part of teaching, T1 said "teachers should let their students sense the nature of mathematical knowledge, and it is less important for them to replicate well-known mathematical formulas" (T1, 971117, after mathematical induction). In addition, there were at least six protocols found to be relevant to this major principle of pedagogical selection and judgement. Two examples were:

(T1, 971117, after mathematical induction)

T1: ---Meanings and concepts play critical roles in students' long-term learning and memory ---and giving my credits and values to the former is not to say that the latter is of no use in teaching and learning. It is only my personal preference to value and use the former but not the latter.

(T1, 971215, after mathematical induction)

T1: ---You have to let your students realize that *the nature* and the content of mathematical knowledge are *real and approachable, and open for investigation*--not just for endless abstraction.

T1: As I said before, it is very important for the students *to learn and make sense of the essence*, rather than the forms, of the mathematical knowledge.

T1: ---The mathematical formulas are useful in communication, but *their nature is critical for making sense of the knowledge.*

B. *Mathematics teaching seeks to motivate students' interest and willingness to learn:*

The starting metaphor and the activity of Hanoi Tower were used to develop students' feelings and impressions concerning the concept. As referred to in the teacher's talks about the "King's Birthday Party" metaphor, we find that values of the concepts of "infinity" and "infinite deduction" relating to the underlying concept of mathematical induction were implicitly transmitted during the following dialogue:

(T1, 971021, video transcript)

T1: Once upon a time there was a kingdom. At the King's birthday party, all the people came to celebrate. Someone said "*Long live the King*", while others prayed for "*His eternity*".

T1: One of the people proposes the phrase "*We wish the King still sees tomorrow's sunrise*". -----.

T1: The King was shocked ---. Yet, it seems not so bad if he can see sunrise everyday?

T1: This story of "*seeing the sunrise forever*" has to do with the concept of "mathematical induction" that we are going to learn today.

In the lessons after interviews typically focused on this aspect of teaching, T1 said that the nature of mathematical knowledge, such as the ideas underlying the metaphor (King's Birthday Party) and the activity (Hanoi Tower), should be used and transformed in teaching "to encourage students to do mathematical investigations in which pleasure in and enjoyment of knowledge are of paramount importance" (T1, 971117, after mathematical induction). Furthermore, there were at least four protocols found to be relevant to this major principle of pedagogical selection and judgement. Two examples of these were:

(T1, 971117, after mathematical induction)

T1: I really hope that all of my students will *feel happiness, enjoyment, and pleasure* in their own processes of investigating knowledge during this activity. This also points out *the affective and* *humanistic concerns* that I have been trying very hard to express through my teaching mathematical knowledge, such as the section on mathematical induction.

(T1, 971215, after mathematical induction)

T1: I hope that my students are able to *explore or investigate any possibility* on their own, and not just by following my steps.

T1: Students should be encouraged to explore mathematical knowledge by following their *naive thinking*.

In the interviews which focused on the lesson planning for the activity, T1 said "I intend to develop an activity in which my students will get the feeling that mathematics is very interesting and they will attend the following lessons" (T1, 971013, before mathematical induction). The Hanoi Tower activity was designed to motivate each student through learning the concept of mathematical induction. One pedagogical value that T1 intended to transmit through the "Hanoi Tower" activity was "*Mathematics teaching is an activity to increase students' motivation and anticipation for learning*." He referred to this major principle of pedagogical selection and judgement twice during the interviews. An example was:

(T1, 971013, before mathematical induction)

T1: I realize that most students are *not happy* during the mathematics lessons, even though their achievements are good on tests. Why? Because the mathematics teaching is *not human* (there are lots of formulas and rules to memorize) and *the students don't feel that the knowledge is useful or practical in their life*. Therefore, most of them feel panic and anxious when learning mathematics. This is the reason I have been trying so hard to motivate them to learn mathematics through *enjoyment, pleasure, and anticipation* using investigative games or activities, and focussing on *the nature of the knowledge*, in order to summon them back.

T1: Teachers should use knowledge itself to motivate students' *willingness and eagerness to learn mathematics*. Let them anticipate that the next day's lesson will have certain new knowledge and surprises. Most importantly, you should let them feel that the mathematical knowledge they are going to learn or they are learning is *practical and useful*, and *helpful* as the basis for learning other knowledge later or as a foundation that might be used in practice.

Therefore, in the interviews either before or after the lessons, T1 argued forcefully that "mathematics teaching should guide the students to enjoy, expect, and be eager to learn mathematics, and should touch their heart." He revealed a sense of eagerness, enjoyment, and high expectations towards mathematical knowledge, and said "the objectives of mathematics teaching are to encourage students to do mathematical investigations in which pleasure and enjoyment of the mathematical knowledge are of paramount importance."

As a result, a core pedagogical value that T1 wanted to develop through the "Hanoi Tower" activity was "*Mathematics teaching is an activity to initiate desire, expectations and enjoyment of knowledge*." There were eight protocols found to be relevant to this core principle of pedagogical selection and judgement. Two examples were:

(T2, 980427, after permutations)

T1: ------ After self-reflections throughout the years, what I am most concerned about in mathematics teaching is the *affective aspects*. ------ I hope that my students can have *positive feelings towards mathematics*, even though they might fail tests or have low achievement. ------ If we can just let them feel that mathematics is somehow useful and interesting, perhaps one day they will be happy to learn it again. So, let these feelings always stay with them and make *the knowledge approachable and worthy of learning*.

T2: ------ When some students are not very confident or happy with mathematics, I am very upset.

T1: So, it this very important for you too.

I: Is this easy to say but difficult to do?

T1: I think this as to do with teachers' personal attitudes and values with regard to mathematics and teaching. If the teacher only expects to teach students skills for solving mathematics problems for tests, then the teacher-student relationship will be quite different from mine, where I hope to *share something interesting with my students, something useful, and fun*--and not only for passing examinations.

T1: I would like to use a "*show*" as a metaphor for mathematics teaching, which needs to convey *tensions entailing attractions and expectations* to the audience. We also need to get our students to feel that *learning mathematics is interesting and worthwhile*, and let them feel *eager to know the remaining parts and the ending* of the topics they have been involved with.

T1: ------ I hope, if I can, to build students' confidence in mathematics from the very beginning. I will try to let them know that *mathematics is acceptable and approachable and give them a sense of liking and eagerness to learn.*

(A, 980511, mathematical induction)

T1: ---A lesson without *surprises* and *expectation* to search for new knowledge is *no fun*, and learning mathematics should include *investigating mathematical knowledge with pleasure*. It is basic human instinct to discover knowledge in order to fulfill personal needs and interests.

Mathematics teachers should establish a learning environment based on students' needs and experiences. As T1 said, "You have to construct an environment or a context for learning in which the mathematical knowledge is well situated, and then it will bring forth the needs of learning." (T2, 980330, after permutations). The mathematical problems should then be analyzed in different situations by developing new tools to solve them. "We should use real problems to direct student thinking, and let them realize that their present knowledge is not enough to deal with such problems; therefore we need to search for new tools to solve the new problems." (A, 980504, mathematical induction). Teachers need to give their students the feeling that they are solving the problem together. What teachers can do is just to "attract them and get them to like the knowledge that they are investigating." (A, 980525, mathematical induction). In addition, "It is very sad for me when my students confront this knowledge without pleasure." (B, 980601, functions).

Moreover, T2, ST1, and ST2 (the other 3 independent observers) also had the same impression of T1 (Chin & Lin, 1999). This teacher tried to stimulate student enjoyment and eagerness in the investigation of mathematics knowledge, through which their knowledge, abilities, and intellect could be developed. These values influenced what he thought about mathematics teaching and how he acted in the mathematics classroom. These values, as a result, formed a system of principles for thinking and acting inside and outside of the mathematics classroom.

3. Pedagogical Value System

The five pedagogical values relegated to the self phase, as shown in Table 2, are the most central and salient principles T1 used to make relevant pedagogical choices and judgements. When he thinks beforehand about his classroom teaching, these values reshape into the six relative values attached to the intention phase. Through pedagogical reasoning (Brown & Borko, 1992), the T1 then transformed these intended values through certain teaching activities, such as the Hanoi Tower, into the nine relative values attached to the implementation phase. For example, in the pedagogical component, the central principle used by T1 when considering mathematics teaching was "Mathematics teaching is an activity for initiating desire, expectation, and enjoyment of knowledge." He then transformed this principle into an intention to realize the principle expressed as "Mathematics teaching is an activity for inducing student motivation and anticipation of learning." As a result, he taught mathematics in the classroom using the two major principles "mathematics teaching seeks to teach students the nature of mathematical knowledge rather than mathematical formulas" and "mathematics teaching seeks to motivate students' interests and willingness to learn."

It seems to us that T1's pedagogical value system, as represented in Fig. 1, is "a holistic knowing and acting unity" consisting of several individual and salient pedagogical values. This system acts as the supreme principle activating the teacher's pedagogical intentions and shaping his classroom teaching activities, in which mathematics is recognized as practical, useful, and interesting knowledge. The valuing self inside the tetrahedron consists of five aspects. For T1, the social facet derives from an observation of human evolution. The educational facet is related to the humanistic concerns of education. The mathematical facet results from a perception of the nature of mathematical knowledge. The mathematics educational facet derives from a sensitivity toward mathematical abilities. Finally, a vision of the nature of mathematics teaching creates the pedagogical facet.

The system starts from the teacher's experiences and reflections on mathematics teaching (the pedagogical facet), and then connects with a recognition of mathematics education (the mathematics educational facet). This is incorporated into an understanding of the concern of education (the educational facet) and the nature of mathematical knowledge (the mathematical facet). Finally, the teacher's pedagogical thinking goes into the concerns of society (the social facet). The content and mechanism this valuing self serves as the implicit and core principles for pedagogical thinking, decisions, and actions.

IV. Reflections on Research on Teachers' Pedagogical Values

In this section we re-think and reflect on the methodologies used, the interpretative methods adopted, and the difficulties confronted in this study of mathematics teachers' pedagogical values. These reflections might be useful in designing a follow-up study, and our experiences may also be helpful for researchers who are interested in investigating teachers' values.

1. The Research Methods

The pedagogical values were explored through a "Four-Step Dialectical Procedure," in which each step explored different features of a teacher's pedagogical characteristics. In the step of Observation and Sensation, we focused on some of T1's observable teaching patterns or teaching styles. These included the metaphorical story "the King's Birthday Party", which started the mathematical induction lesson; and the follow-up activity of "Hanoi Tower" described earlier. Some personal characteristics of the teacher were also noted, such as "sharing and discussion" and "using-metaphors," as described by the other observers:

(A, 980525, mathematical induction)
(T2, ST1, ST2, 980119, final meeting of the 1st semester)
T2: I just sat in front of him (T1) ------ I have observed him almost every day for more than two years, but not on purpose of course. It seems to me that he has a kind of natural spirit and sense in teaching, and for being a mathematics teacher. For example, he always likes to *share and discuss things that he understands* with colleagues at the school. He is confident in what he is talking about, and is *a teaching enthusiast* who loves being a mathematics teacher.
(T2, ST1, ST2, 980119, final meeting of the 1st semester)
(T2, ST1, ST2, 980608, final meeting of the 2nd semester)

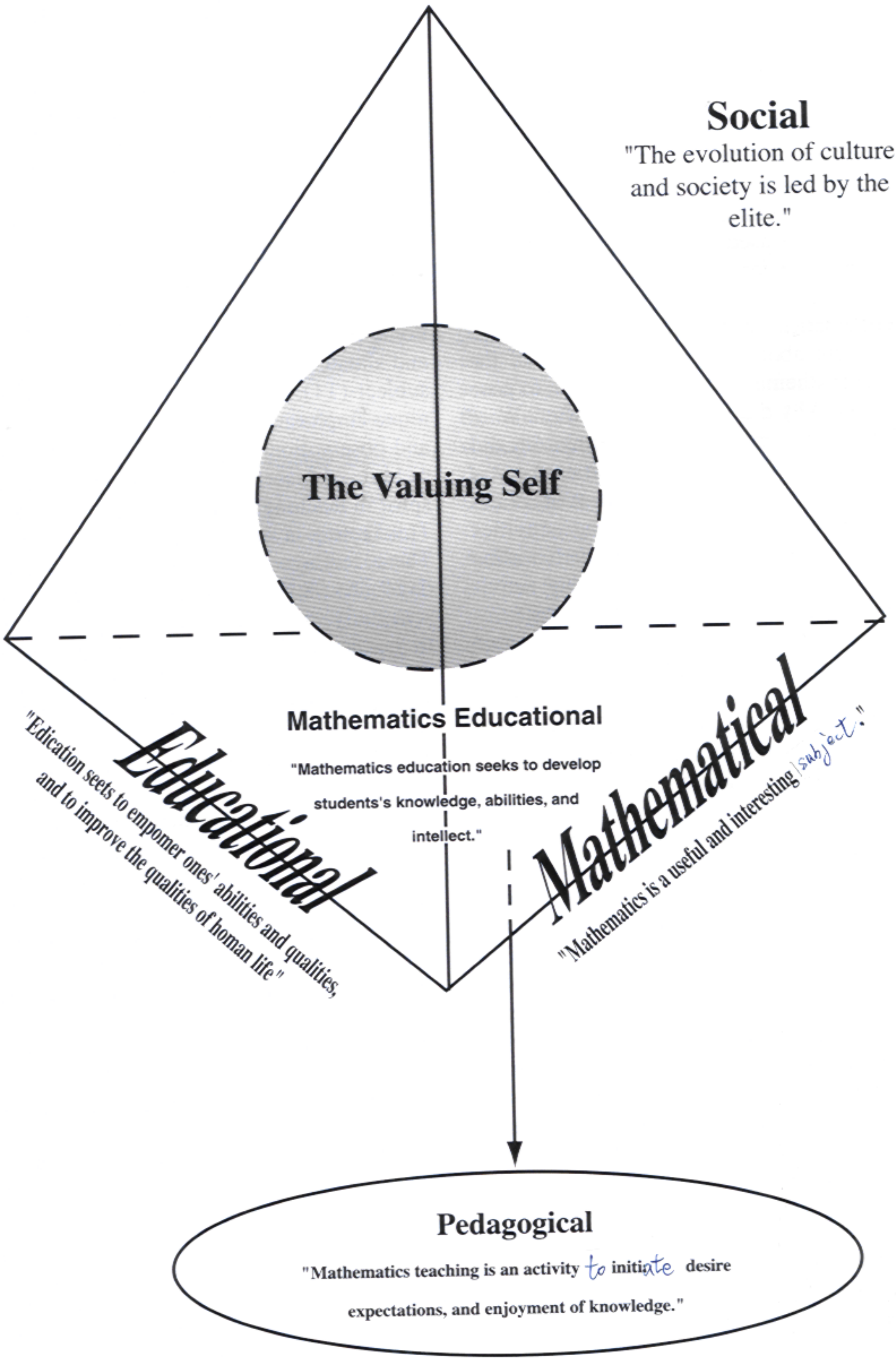

Fig. 1. A Representation of T1's Pedagogical Value System.

ST1: He (T1) always tries to connect mathematical concepts or knowledge with practical applications. He then tries to figure out the ways of introducing the concepts at the very beginning. He also uses some metaphors or real examples to uncover and relate the students' experiences and pre-conceptions as the sources and domain for application.

A teacher's teaching styles and characteristics might point to that teacher's value-derived phenomena.

During the second step, the processes of Reflection and Introspection were used. It is thought to be critical to induce the teacher to reflect on his own teaching activities. We used "pedagogical reflections and introspection" as a form of interview to uncover teacher's knowledge, thoughts, or identifications regarding mathematics and pedagogy. When the teacher had thought about his own teaching and the ways of teaching mathematics, then it was time to pose such questions as "Why did you teach that way? Were there any other alternatives? And for what reasons did you choose them?" or "Why is it so important for you to teach mathematics that way?" The content of reflections then shifted from T1's own teaching activities involving preactive, inter-active, and post-active ideas and reflections, to a colleague's (T2) teaching activities, and finally to the four exemplary mathematics teachers (A, B, C, D). This formed a better climate and situation for reflections through a "intra-personal process" (Canning, 1990; Sikes & Aspinwall, 1990). This "self-to-others pedagogical reflection and introspection" approach focusing on the situated pedagogical events observed was useful in uncovering a teacher's values.

Values are conceived as personal principles or standards for choosing and enacting certain pedagogical identifications, and the principles or standards for thinking and acting. Dialogue and Discussion were used to verify these principles or standards. "Value dialogue" was critical during the interviews, since this brought out teachers' rationales or principles concerning values. The discussion approach was open and informal, and issues were clarified in a way similar to the approaches of "value clarification discussions" (Volkmor, Pasanella, & Raths, 1977) and "thinking aloud" (Clark & Peterson, 1986). In such dialogues, the researcher acts as a listener and inquirer in examining whatever he recognizes as relevant; the teacher, on the other hand, acts as a speaker in recollecting thoughts and principles of judgement through introspection and retrospection on his teaching activities. It is a dialectical and discursive process in which the teacher may try to defend his positions or rationales for selection and judgement through mutual exchanges.

In the final step, a Recursive Probing approach conforming to the previous three steps and a multi-faceted triangulation were used to identify T1's pedagogical identities. Empirical data referred to in this study were examined and verified in several different aspects and contexts. For instance, one colleague (T2) and two student teachers (ST1, ST2) were involved in collecting and re-examining the data. The data, however, was collected from several different sources, such as T1's regular interviews, his in-school colloquiums, and out-of school conversations. The focus of data collection was therefore on the teacher's daily teaching. The major difference between this "multi-faceted triangulation" and the traditional triangulation technique was "who is the focus?" Our approach stresses situational understanding (Elliott, 1993; Hargreaves, 1993) and situated knowledge (Lave & Wenger, 1991) to the subject (T1). On the other hand, the traditional technique emphasized the researchers' experimental design and observations. As T2 described, "I just sat in front of him ---! It seems to me that he has a kind of natural spirit in teaching, and for being a mathematics teacher. This feeling is strong and real, but not for this project. These feelings become stronger and stronger everyday." (A, 980525, mathematical induction). The present study stresses the social nature of values that seem to be personal, situational, and experiential (Raths, Harmin, & Simon, 1987; Stewart, 1987; Swadener & Soedjadi, 1988).

2. Value Interpretation

The pedagogical values were identified by following a "Four-Level Interpretive Phase", in which each step focused on different teacher characteristics. In the primary phase, some Value Phenomena such as observable or sensible teaching styles and personal characteristics were described. Relevant Value Indicators such as pedagogical knowledge, thoughts, or identifications were examined. The Value Candidates concerning the principles of selection and judgements, were then brought out. Finally, these value candidates represented the Values (pedagogical identities) of the teacher.

A process for examining teachers' pedagogical values through a "Phenomena-Indicator-Candidate-Value" format has thus emerged. Fig. 2 illustrates a methodological framework for investigating and interpreting mathematics teachers' pedagogical values. The phenomena of values revealed specific features of teaching and the teacher's personality that have become apparent and observable by outsiders, such as the researchers T2, ST1, and ST2. The emerging teaching events then acted as catalysts for examining the knowledge or pedagogical identifications relating to observ-

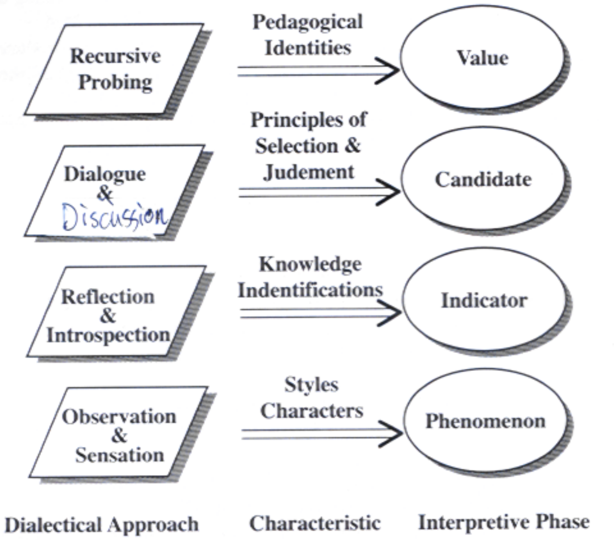

Fig. 2. A Framework for Investigating and Interpreting Teachers' Pedagogical Value Systems.

able and sensible phenomena, pointing to the principles or standards of choosing this knowledge or identifications which might be used to locate the pedagogical identities of a mathematics teacher. The principles that T1 valued had been developed during a long-term process of self-reflection, selection, and judgement, as shown in his recollections of a twenty-year teaching career and his views on other teachers' (A, B, C, D) teaching activities (Chin & Lin, 1999). Therefore, the consistent pedagogical values that were intended and implemented in the mathematics classroom were the result of thoughtful consideration of the consequences. This refers to the "Choosing" and "Acting" procedures of valuing (Raths, Harmin, & Simon, 1987).

In light of the choosing and acting procedures of valuing, we concluded that a teacher's pedagogical values can be defined as "principles for selecting and judging certain pedagogical identifications on the basis of whether they are of importance or worth to his or her classroom teaching of mathematics." Pedagogical values are therefore conceived as personal principles of thinking and practicing certain pedagogical identities with which the teachers agree. Personal identity refers to those qualities and characteristics we see in ourselves (Augoustinos & Walker, 1995), and therefore stresses individual differences. On the other hand, social identity is defined as an individual's self-concept which derives from the knowledge of membership in a social group or groups with the values and emotional significance of that membership (Tajfel, 1978, 1981), which therefore emphasizes commonality across individuals. Based on the analysis by Cooley and Mead, Jenkins (1966) who argued that individual and social identities are constituted through an internal-external dialectic of identification.

Therefore, in this study, we conceived the values that mathematics teachers have as their "Pedagogical Identities" concerning mathematics and pedagogy, developed through a dialectical relationship between the varieties and complexities of individual pedagogical identifications. They are the results of a process of an internal-external dialectic of identification. We consider the teacher to be a member of the teacher group to which he belongs. His pedagogical values then reflect his principles of selection and judgement with regard to certain identities concerning mathematics and pedagogy, which are shared among group members (teachers). These identities describe not only teacher's personal characteristics of mathematics teaching, but also the shared characters of the teacher group. These shared identities reflect much of the specific features that mathematics teachers have when teaching within their context of schooling.

3. Research Difficulties

Pedagogical values (identities) are part of each teacher's personality, and involve an individual's recognition and beliefs concerning education, mathematics, and pedagogy. The implicit nature of values or identities created many obstacles. We were often forced to modify our interview procedures and strategies to a more recursive and reflexive approach based on the teacher's responses and his willingness to talk. Moreover, to discuss teaching issues with a teacher who has taught mathematics for 20 years, the researchers felt more like listeners and learners rather than experts. This friendly relationship may encourage teachers to share their ideas in a more comfortable manner.

Another issue was related to the difficulty of differentiating value indicators and values. The former may be construed as attitudes, interests, feelings, or beliefs (Raths, Harman, & Simon, 1987). However, values are supposed to be personal principles or standards for selecting and judging among varied value indicator alternatives. Finally, although introspection and retrospection seemed to be helpful in collecting the teacher's processes of choices or judgements in the past, the limitations of the data were clearly derived from the nature of personal reflections in the absence of more objective verification.

V. Implications

Pedagogical preferences, judgements, and selections influence much of the content taught and the reasons a mathematics teacher prefers to use certain items of knowledge rather than others in his teaching. Values that we described in this study were conceived as teachers' principles for making pedagogical judgements. They are essential for mathematics teaching because of their implicit and powerful role in teachers' thinking and decision-makings. The mechanism of the value system described the logic and thinking patterns of individual values. It also consolidates each teacher's thinking and practices. These interpretations might be useful in developing relevant courses to educate pre-service mathematics teachers in values.

The authors in this paper have re-considered and re-examined the research methods and the approaches to interpreting pedagogical values through self-reflection and self-introspection. Another issue has to do with both changing teachers' pedagogical values and educating them about new values. Although many studies have presented the success of changing student teachers' pedagogical beliefs (e.g., Chin, 1995; Thompson, 1992), it seems difficult to change one's personality or identity in the short term, due to the private and implicit nature. This issue should therefore be examined more carefully.

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一位數學教師教學價值的個案研究：運用方法學架構的詮釋與省思

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摘 要

價值的數學教學是個一直被忽視而且尚待開發的數學教育研究領域。研究文獻報導了數學教師信念的內涵、其教學上的實踐、及對學生學習數學的影響。但是，也反映出數學教師意圖的和實踐的教學信念間的落差。這種落差似與教師內心更深層的教學價值有關。本研究採用個案調查法，透過問卷、教室觀察、和對話式晤談，探究一位資深數學教師的教學價值。同時，也對研究教學價值的方法作反省。

本文透過自我、意圖、和實踐三個面相及社會、教育、數學、數學教育、數學教學五個成分，詮釋這位教師的教學價值及教學價值體系。結果發現在自我、意圖、和實踐三面相中，各有五、六、九種凸顯的教學價值。其中，自我面相有五種結合教學意圖和教學實踐的核心原則。投射到數學教學的情境時，則反映出六種意圖面和九種實踐面的教學價值。這個“價值化的自我”是這位教師思考與實踐某些教學認同的最高準則。

關於研究法的省思，本文建議透過一些精密的研究設計，例如教學反思、現場理解、多面相三角校正、價值對話、遞迴試探法等，從分析教學價值的“現象”著手，找出相應的教學價值“指標”，進而檢驗出相關的教學“價值”。在運用這個“四階段多面相探測”進程時，應配合“說做齊一”和“愼選”兩個檢驗程序，以辨識價值、指標、和現象。藉助社會身份的理論，作者視教學價值爲數學教師的一種教學身份。它是主導教師選擇與判斷不同教學認同的重要性與否或值不值得用於數學教學的準則。而教學價值體系則爲“教學身份的多樣面相和複雜之間的一種辯證相依的互賴關係”。瞭解數學教師教學價值的內涵及其運作的方式，將有助於國內研究者領導此一教師思維新領域的相關研究。此外，本研究結果亦可用於開發教學價值的師資培育課程。